Complex groundwater whirl systems

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Abstract By analyzing three-dimensional flow patterns in finite-element models of layered anisotropic aquifers, we found that streamlines often have the shape of spirals. A bundle of such spiralling streamlines was termed a 'groundwater whirl'. Several analytic solutions have been developed that confirm the existence of groundwater whirls in anisotropic layered aquifers. Further experiments include aquifers in which all layers have a laterally heterogeneous anisotropy. In box-shaped aquifers with horizontal layers and a uniform horizontal gradient along the boundaries, all whirls have their axes in the same flow direction. In such cases projected streamlines can be represented by stream function contours. It allows an easy 2D interpretation of the main characteristics of complex whirls patterns. Clockwise and counter-clockwise whirl axes occur at the interfaces of adjacent layers with different anisotropic hydraulic conductivities. One or more clockwise whirls may occur within one counter-clockwise whirl, and vice versa. Where contours of different whirls meet, saddle points are found, both within the model and on its boundaries. There is a fixed relation between the number of whirls and the number of saddle points. As a consequence of groundwater whirls, the exchange of water between aquifer layers is intensified. The spreading of dissolved substances increases as a result of variations in the directions and magnitude of horizontal anisotropy, and may form an important component of the mechanical dispersion.

Key words saturated groundwater; layered aquifers; heterogeneity; anisotropy; analytical model; streamlines; stream function; groundwater whirls; mechanical dispersion

INTRODUCTION

Anisotropy of the hydraulic conductivity is a well-known concept in hydrogeology. From a sedimentological point of view anisotropic properties with spatial variations are more likely to occur than isotropic conductivities. However, the vast majority of groundwater models approximate the hydraulic conductivity as isotropic, probably because little or no information is available on the magnitude and variation of the anisotropy. Little is known about the errors associated with this simplification. Insight into the magnitude of the error may be gained through the study of hypothetical cases. There is a growing number of finite-difference (e.g. MODFLOW) and especially finite-element model codes (e.g. FEFLOW, MicroFEM, SUTRA) that can handle three-dimensional variations of the anisotropic conductivity. Hemker et al. (2004) started experiments with some simple numerical models consisting of anisotropic blocks in an otherwise homogeneous and isotropic confined aquifer. Three-dimensional streamlines in such models have the shape of spirals; bundles of spiralling streamlines rotating in the same direction were termed 'groundwater whirls'. Further investigations deal with increasingly complex hypothetical models of stratified aquifers (Hemker & Bakker, 2004). Most numerical models were built with the finite-element model code MicroFEM (MDS, 2005). New analytic solutions were developed for flow in layered anisotropic box-shaped aquifers (Bakker & Hemker, 2004). A comparison of the flow in analytical and numerical models shows the same whirling groundwater flow patterns as a result of stratification and anisotropy (Hemker & Bakker, 2004).

The objective of this paper is to present typical whirl patterns that can be identified in relatively simple models of layered anisotropic aquifers. In these models all layers have a uniform thickness. The hydraulic conductivity is vertically and laterally heterogeneous, but only the major principal direction of the horizontal conductivity varies spatially in a plane perpendicular to the general flow direction; the principal values of the conductivity are the same throughout the entire model.

The vertically and laterally discontinuous horizontal anisotropy leads to complex patterns of clockwise and counter-clockwise groundwater whirls. The practical consequence of these whirls is that water is exchanged between aquifer layers, even when the general direction of flow is parallel to the layered structure of the aquifer. This may have a significant impact on contaminant spreading throughout the aquifer.

ANALYTICAL MODELLING

New analytic solutions for groundwater whirls in layered anisotropic aquifers have been developed recently for a box-shaped aquifer. The aquifer consists of a number of horizontal layers, and its width is subdivided into a number of strips. In this way the aquifer is divided in many homogeneous blocks that extent over the entire length of the aquifer. The general flow direction is in the length-direction of these blocks. An analytical model may consist of thousands of such blocks with different anisotropic hydraulic conductivities.

In this paper results are presented from several models with the following general setup. Consider an 18 m thick confined aquifer, consisting of nine equally thick layers. The horizontal hydraulic conductivity is heterogeneous in a 100 m wide section of the model; each layer in this section is divided in 10 strips of equal width. A cross- section perpendicular to these strips shows a regular pattern of 9-by-10 cells, where each cell is 10 m wide and 2 m high (Fig. 1). On each side of this central zone a 100 m wide homogeneous block serves to reduce boundary effects. The east and west sides are no-flow boundaries, while the south and north sides are open boundaries with a fixed flux.

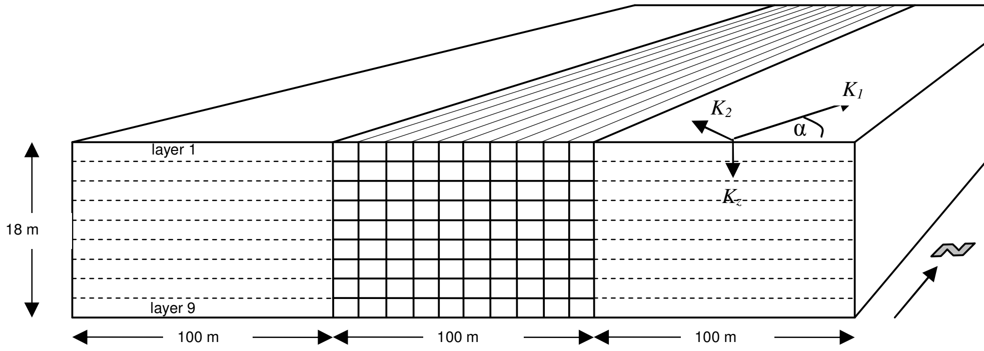

Fig. 1 A stratified confined aquifer with a laterally heterogeneous anisotropic central zone.

The major and minor principal values of the horizontal hydraulic conductivity tensor are 10 m day^{-1} and 5 m day^{-1} in the entire model. The vertical hydraulic conductivity is 1 m day^{-1} in all layers. Due to the specified boundary conditions, the general flow direction is in the direction of the strips (straight north). The major principal direction of the horizontal hydraulic conductivity tensor is also chosen straight north in the two large side blocks, while it varies in the 90 cells of the central zone. By choosing different distributions of principal directions for the central zone, a large number of models was constructed and analysed. Some of these models were used to compare results with finite-element models (Hemker & Bakker, 2004).

BASIC FLOW PATTERNS

If a model is isotropic (or if all principal directions are the same) all streamlines flow straight north and the projection of a single streamline, when looking downstream, is a single point. As an example of the flow patterns in case of spatially varying anisotropy, ten three-dimensional streamlines in a heterogeneous anisotropic aquifer model are shown in Fig. 2. The variation of the principal directions is shown in Table 1.

Table 1 Principal directions of horizontal anisotropy in all 9 by 12 cells of the model shown in Fig. 2.

90	170	170	170	170	170	50	50	50	50	50	90
90	180	180	180	180	180	40	40	40	40	40	90
90	190	190	190	190	190	30	30	30	30	30	90
90	200	200	200	200	200	20	20	20	20	20	90
90	210	210	210	210	210	10	10	10	10	10	90
90	220	220	220	220	220	0	0	0	0	0	90
90	230	230	230	230	230	-10	-10	-10	-10	-10	90
90	240	240	240	240	240	-20	-20	-20	-20	-20	90
90	250	250	250	250	250	-30	-30	-30	-30	-30	90

To assist in the interpretation of the streamlines, two-dimensional projections are added on the horizontal plane and the two vertical planes. Especially the projection on the vertical plane normal to the north direction (the back plane of Fig. 2) aids in the interpretation of the flow field. It may be concluded from this projection in Fig. 2 that there are at least one counter-clockwise whirl (on the left) and one clockwise whirl (on the right).

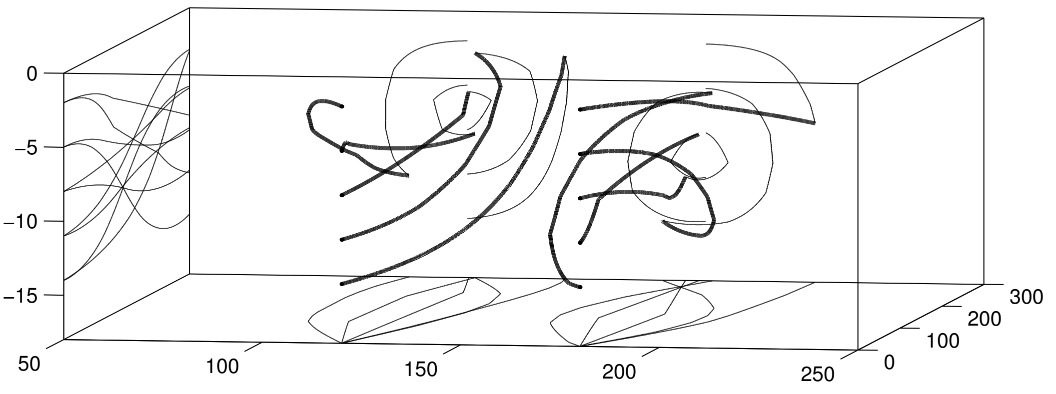

Fig. 2 Ten three-dimensional streamlines (thick) and their projections on horizontal and vertical planes (thin). Streamlines start at the front at 120 and 180 m from the left model boundary and at depths of 2, 5, 8, 11 and 14 m to identify a flow pattern of one counter-clockwise and one clockwise whirl.

In the remainder of this paper, whirl patterns are identified by studying the projection of three-dimensional streamlines on this vertical cross-section normal to the general flow direction. These projected streamlines may be described with a two-dimensional stream function (Bakker and Hemker, 2004). A detailed projection of the three-dimensional flow pattern of Fig. 2 is shown in Fig. 3b. From this figure it is clear that there are exactly two whirls, rotating in opposite directions.

In complex whirl systems it was found that several whirls may occur within one larger whirl, all rotating in the same direction (Hemker & Bakker, 2004). Within these whirls even smaller ones may be present. It is also possible that within a larger whirl, one or more whirls occur rotating in opposite directions. Some of these basic flow patterns are presented in Fig. 3.

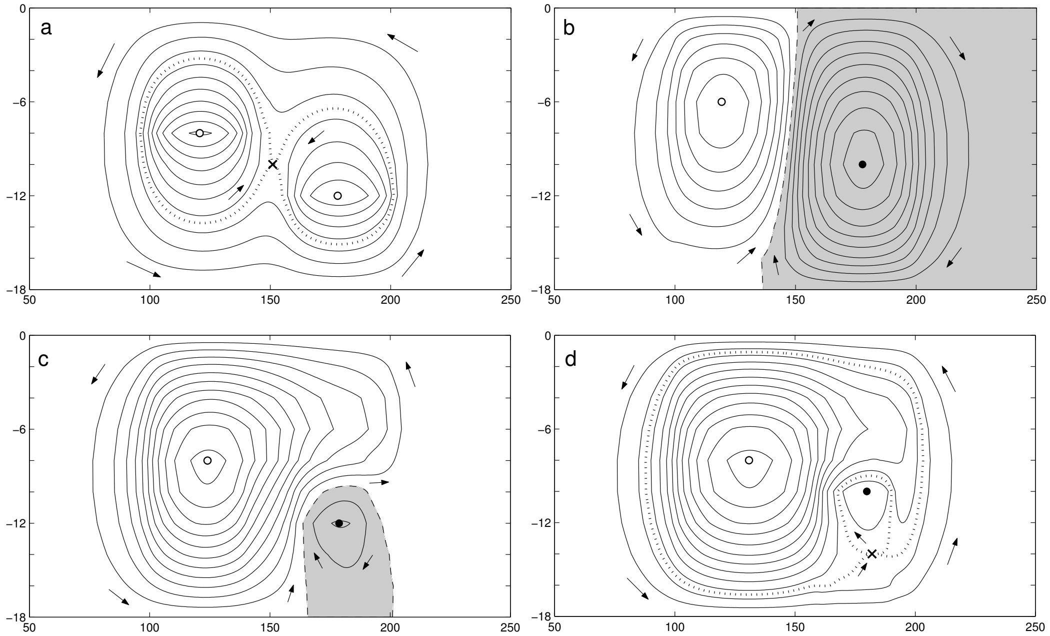

Fig. 3 Examples of projections of simple three-dimensional flow patterns, showing the axes of clockwise and counter-clockwise whirls (● and ○), saddle points (x-mark), negative and positive stream function areas (white and grey) and whirl boundaries (dotted or dashed line): (a) two whirls rotating in the same direction, and (b), (c) and (d) two whirls rotating in opposite directions.

The streamline patterns in Fig. 3 all have two whirls that rotate in the same (a) or in opposite directions (b, c and d). Two whirls rotating in opposite directions meet in a single point, which is a saddle point of the stream function. Note that a saddle point is a two-dimensional representation of a specific three-dimensional streamline on the boundary of the whirl along which the flow is solely in the north direction. The dotted lines depict the streamlines bounding the whirls. Streamlines started on different sides of these boundaries are part of different whirls; they will diverge significantly but will eventually come back to the same spot (in the projected plane). A whirl boundary may be connected to the model boundary (Figs. 3b and c). In such cases the stream function value along the boundary of the whirl is equal to the stream function along the boundary of the aquifer, which simplifies the identification of the whirls.

When a whirl is fully enclosed by another whirl rotating in the opposite direction (Fig. 3d), there is a saddle point on the whirl boundary. The stream function at this boundary will, in general, not be zero. The boundary of the enclosed whirl is formed by the streamline that touches the saddle point.

COMPLEX FLOW PATTERNS

The flow patterns of Fig. 3 were created by choosing relatively simple spatial distribution patterns of the major principal anisotropy direction for the 9-by-10 cells in the central part of the model. When the anisotropy directions vary more randomly, complex flow patterns are obtained. Hemker & Bakker (2004) present a model, based on ten uniformly distributed anisotropy directions between 45° (northeast) and 135° (northwest) assigned to each layer in a random order (Table 2). The resulting flow pattern is given in Fig. 4. The stream function varies between -0.0039 (white) and 0.0045 m^2 day^{-1} (dark grey). Whirl axes are indicated by 16 white and 18 black circle marks. The position of these axes is found by inspecting all locations along the layer interfaces where the vertical and horizontal flow components change direction.

Table 2 Principal directions of horizontal anisotropy in all cells of the model shown in Figs. 4 and 5.

90	125	85	75	55	105	115	95	65	135	45	90
90	65	55	125	75	135	95	115	85	105	45	90
90	125	65	105	115	95	75	85	135	45	55	90
90	125	55	85	115	75	95	45	65	105	135	90
90	75	95	65	135	55	45	85	125	105	115	90
90	135	125	65	115	85	105	75	55	95	45	90
90	65	135	95	55	115	85	75	105	45	125	90
90	55	85	105	125	65	135	115	75	45	95	90
90	45	115	125	95	105	75	85	135	55	65	90

Dashed lines indicate the boundaries of whirls rotating in opposite directions. Some of these lines run from the top to the base of the aquifer and separate the larger clockwise and counter-clockwise whirl systems. Each of these whirl systems are composed of many smaller whirls rotating in the same direction; in some cases, they also contain smaller whirls rotating in the opposite direction. Other individual whirls are attached to the impervious top or base of the aquifer.

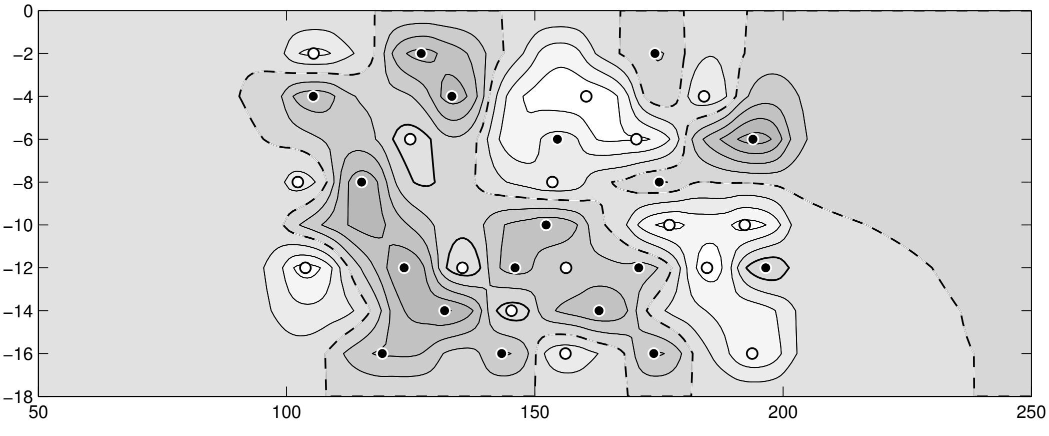

Fig. 4 Stream function contours in a cross-section of the aquifer. Zero-valued contours (dashed lines) separate counter-clockwise (light grey) and clockwise whirls (grey). Whirl axes are marked by ○ and ●.

A more complete view of the complex flow pattern can be obtained by also plotting the saddle points (Fig. 5). These points may occur at layer interfaces as well as within the layers. Altogether 28 saddle points were found within the model, of which 6 were used to delineate the boundaries of enclosed whirls rotating in opposite directions. The number of saddle points is found to be the same as the total number of whirl axes minus one, provided that each pair of intersection points of whirl boundary and model boundary (indicated by ◊ symbols in Fig. 5) is counted as one saddle point: $18 + 16 - 1 = 28 + (10/2)$.

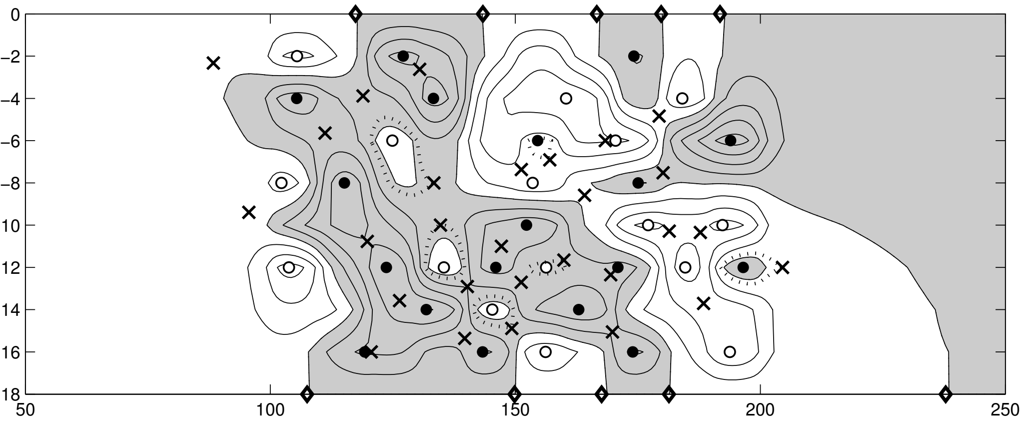

Fig. 5 Same flow pattern as Fig. 4 showing all saddle points (x), boundaries of enclosed whirls rotating in opposite directions (dotted) and locations where whirl boundaries meet the model boundary (◊).

In the whirl patterns visualized by stream function contours, three different types of critical points can be distinguished: the whirl axes (● and ○), the saddle points (x) and the intersection of whirl boundaries with the model boundary (◊). Only streamlines flowing through these critical points are straight lines, while all other streamlines rotate around one or more axes, exchanging water between two or more adjacent layers. From the patterns of Figs. 4 and 5 it is clear that a small part of the total flow follows complex-shaped streamlines between top and bottom, passing all layers of the aquifer.

A practical consequence of groundwater whirls is that the exchange of water between aquifer layers is intensified. This increased lateral and vertical mixing is an advective process (mechanical dispersion) caused by variations of anisotropy in a layered aquifer. It may have a significant impact on contaminant spreading.

CONCLUSIONS

Recently developed analytic solutions were used to build models of laterally heterogeneous anisotropic aquifers. Plotting stream function contours facilitated the analysis of the resulting complex patterns of 3D spiral-shaped streamlines. Clockwise and counter-clockwise groundwater whirls have their axes at the interfaces of adjacent layers with different anisotropic hydraulic conductivities. One or more clockwise whirls may occur within a counter-clockwise whirl, and vice versa. Where contours of different whirls meet, saddle points are found. There is a fixed relation between the number of whirls and the number of saddle points. A practical consequence of groundwater whirls is that the exchange of water between aquifer layers is intensified, which may have a significant impact on contaminant spreading.

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