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Journal of Hydrology, 225 (1999): 19-44

### Summary

## GENERAL INTRODUCTION

### 1. INTRODUCTION

In the Netherlands, as in most populated parts of the world, groundwater constitutes an important component of water supply for domestic use, industry and agriculture. This groundwater is preferably withdrawn from aquifers, i.e. geologic formations that are capable of yielding significant amounts of water. As a result of pumping, the water table and the water levels in nearby wells will be lowered. The demand for water usually grows and at some stage the withdrawal of groundwater may cause undesirable side effects. Such adverse situations can be prevented or reduced by a good management of water resources. This requires insight into the managed hydrological system in general and the ability to forecast the response of the aquifer system to proposed pumping. The usual tool for analysis and prediction is a numerical model that simulates regional groundwater flow. The construction of a groundwater flow model requires sufficient information about the hydraulic properties of the subsurface. In general, pumping tests provide the most reliable information to determine the hydraulic properties of aquifer systems.

Apart from groundwater exploitation, specific conditions in the western part of the Netherlands often require abstraction of groundwater for water management purposes. More than half of the Netherlands can be classified as a coastal lowland with a permeable subsurface of 100 - 300 m of Pleistocene fluvial sands, separated by one or more clay layers. This complex aquifer system is confined by a semi-pervious layer of 10 - 20 m of marine and fluvial clayey and peaty deposits from the Holocene. Because abundant open water in ditches, canals, rivers and lakes are present at the surface, the top layer of this extensive leaky aquifer system remains almost completely saturated. This situation poses special problems for building pits, and extensive groundwater abstraction schemes are often needed to keep groundwater pressure and seepage under control in these excavations.

The hydrogeologic conditions in the Dutch lowlands were in fact the main reason in the beginning of the 20th century to investigate well flow under leaky aquifer conditions. Until the 1930s the design of well fields was mainly based on empirical information on the relation between abstraction rate and drawdown, obtained by test pumping. However, as early as 1914, Kooper found the first analytical solution for steady well flow in a homogeneous, single leaky aquifer. Further evaluation of his solution in practice and an extension to partially penetrating wells was subsequently offered by De Glee in 1930 (De Vries, 1982). In the Netherlands the Kooper/De Glee solution is generally referred to as the De Glee formula. The same solution was published in 1949 by Hantush.

A wide range of analysis techniques subsequently became available for single homogeneous aquifers. Few methods have been developed for two-aquifer systems, i.e. two aquifers above each other, separated by a considerably less permeable layer (Kruseman & de Ridder 1970, 1990). However, until twenty years ago no solutions were available for leaky multi-aquifer systems comprising of three or more aquifers. The main objective of the present study is to determine analytical solutions for well flow in multi-aquifer systems, in order to allow for proper analysis of pumping tests in such systems.

This study of groundwater flow in layered aquifer systems was carried out in two separate periods. Flow in aquifer-aquitard systems was investigated during the years 1979 to 1986 and results were published in reports (Hemker, 1981a,b) and in three consecutive journal articles (Hemker, 1984,

1985a; Hemker & Maas, 1987). More recently, the same solution technique was further developed to investigate flow near wells in layered aquifers. This extension of the multi-aquifer solution technique has been described in two papers that are to be published as journal articles (Hemker, 1999a,b). The five interrelated papers are assembled in their logical order and presented here sequentially as Chapters 2 to 6.

The remainder of this introduction gives a further description of the motives, objectives and coherence of the compiled parts. In section 2 a brief account is given of some early experiences with well flow in a multiple aquifer system and the lack of a proper method for analysis of the obtained drawdown data. As discussed in Section 3, a solution was achieved by progressive development of a number of analytical equations for such systems. Numerical tools were subsequently developed for multiple aquifer analysis, as detailed in Section 4. The following section describes how during a new phase of analytical developments, multi-aquifer theory is further adapted for a multi-layer approach to solve well flow problems in homogeneous and heterogeneous aquifers. Section 6 concludes this introduction with a list of applied methods and conceptual well flow model characteristics, in their order of appearance in Chapters 2 to 6.

## 2. BACKGROUND: THE LANGERAK PUMPING TEST PROBLEM

In 1978 the Dutch Water Supply Company "Ablasserwaard en Vijfheerenlanden" started hydrological investigations in search of suitable well field sites in the polder area east of Rotterdam, in order to increase future drinking water production. In this very flat country, with large meandering rivers, groundwater flow is mainly driven by the different water levels of rivers and individual polders, in combination with the leaky conditions of the Holocene top layer. From hydrogeological and geochemical investigations it was clear that on a regional scale four fresh water aquifers were involved. The 10-60 m thick fluvial and marine sand layers are separated and covered by 5-20 m thick clay, loam and peat deposits. Further exploration of the top and second aquifer was planned. A test-hole to a depth of 200 m near Langerak (Chapter 2, Fig. 1) showed that some of the regional aquifers are subdivided locally by intercalated clay layers. Brackish groundwater was only found at depths below 170 m and the location seemed suitable for a water supply well field. A pumping test was planned to provide information on the local hydraulic parameters as a basis for further investigations to quantify the effects of groundwater withdrawal. Piezometers were installed in seven (sub)aquifers in the test-hole, and two pumping wells were drilled at short distances (30 and 41 m) and screened in the second and third aquifers. The pumping test was performed in November 1979 and the pumping period lasted 21 days. The second aquifer was pumped for the full period, while the third aquifer was only discharged during eight days at the end. Drawdowns were measured in all (sub)aquifers and from a visual inspection of the drawdown curves it appeared that steady-state flow was reached in all aquifers within five days from the start of pumping.

The Langerak pumping test showed that the more than 200 m thick series of horizontal aquifers, separated by aquitards, reacted to pumping as a single system. Each aquifer and aquitard is only part of this multiple aquifer system. Much practical insight was gained from the behaviour of this layered aquifer system in response to the pumping test. The pumping test and its analysis were conducted by an external consultant and since there was no method available to analyze this multiple aquifer system as a whole, an approximate solution was developed based on the two-aquifer leaky system solution of Bruggeman (1972), applying corrections for additional drawdowns in the adjacent aquifers (Rolf, 1981).

### 3. A STEPWISE DEVELOPMENT OF MULTI-AQUIFER SOLUTIONS

#### *Motivation*

Following the Langerak pumping test, further hydrological investigations were planned by the "A & V" Water Supply Company, including a large-scale pumping test in the Lexmond well field (Chapter 2, Fig. 1). From drawdowns in the many piezometers near this site it was well known that here too a layered system was pumped and four aquifers should be distinguished. Although the number of aquifers was smaller than at Langerak, analysis of this test would be complicated by the large number (26) of production wells involved and the expected complications caused by induced recharge from the nearby river Rhine.

The Langerak experience, the planned Lexmond pumping test and the lack of a method to compute drawdowns in aquifer systems of more than two aquifers, were the motivation to search for a new equation or a new technique. Just as Langerak produced the background, Lexmond gave the incentive.

#### *Steady-state flow*

An extensive literature study revealed that analytical well flow solutions for two-aquifer systems were occasionally reported, but no work could be found with respect to multiple aquifer systems. By studying publications relating to these two-aquifer systems, especially Huisman & Kemperman (1951) and Bruggeman (1972), and with the mathematical support of P.W. Hemker (my brother), a partial solution was found in December 1979. With this new solution, based on matrix analysis techniques, it was possible to compute the steady-state drawdown at any distance from the well and in any layer of a multiple aquifer system consisting of as many aquifers as required.

The Lexmond pumping test was executed in March 1980. After maintaining a constant minimum discharge for twelve days, all wells were pumped and this produced an additional 550 m<sup>3</sup>/hour. The maximum discharge lasted for 83 hours, after which production was again reduced to a minimum. Estimated values for the steady-state drawdown in all piezometers could be obtained after some extrapolation of the drawdown curves. Because of the large number of parameters involved in multi-aquifer systems, graphical solution techniques are not feasible and a non-linear regression technique needs to be used instead (Hemker, 1985b). The new multi-aquifer solution, the superposition of the many wells involved, the effects of induced river infiltration and the non-linear regression required a special computer program (written in Algol) and many runs on a main frame computer before the final pumping test analysis was obtained. However, the exercise not only produced parameter values for the Lexmond area, but it also demonstrated the practical value of the newly developed multi-aquifer solution. The Lexmond pumping test was reported (Hemker, 1981a) and, subsequently, the same technique was used for a new and better analysis of the Langerak data (Hemker, 1981b). At a later stage the well flow solution was generalized for different steady-state flow systems, including flow from a wide river partially penetrating a leaky top layer, and published in 1984 (Chapter 2).

The same solution for steady-state well flow was subsequently also presented by others (Hunt, 1985, 1986; Maas, 1986; Wu Shi Yu, 1987).

#### *Transient well flow*

Although the developed steady-state solution allowed for proper analysis of the Lexmond and Langerak pumping tests, it should only be regarded as a partial solution for multi-aquifer tests in general. The reason is that pumping tests are usually performed by withdrawing water from one well

and observing the temporal variation of water levels in nearby observation wells. This requires a multi-aquifer solution for transient well flow.

Bruggeman (1972) showed how Laplace and Hankel transforms can be used to derive a solution for transient well flow in a leaky two-aquifer system. The same approach was followed for the non-aquifer case, but now in combination with the previously developed matrix techniques. The resulting analytical solution may be regarded as a generalization of a large number of commonly used single and dual-aquifer solutions for pumping tests, including those of Theis (1935), Hantush & Jacob (1955), Barenblatt et al. (1960), Boulton (1963) and Hantush (1967). The new solution was published in 1985 (Chapter 3). An alternative expression was recently presented by Bruggeman (1999, p. 442).

To enable wider use of the drawdown equations, a computer program was written to analyze multi-aquifer pumping tests (in BASIC, to be run on a microcomputer). It was realized that some limitations could reduce its practical use, and so attention was paid to approximate solutions for storage in aquitards, unconfined aquifers, partially penetrating wells and stratified aquifers.

The determination of hydraulic characteristics (parameter estimation) requires repeated and accurate computation of a large number of drawdowns, which, as a result of the semi-infinite integral representation, needs extensive computer calculations and consequently quite some computer time. Although the main problem was solved, there was still room for improvement.

#### *Well flow with aquitard storage*

Thanks to the demonstration by Moench & Ogata (1984) of the advantages in using the Stehfest algorithm for numerical inversion of Laplace transform solutions, it became clear that transient well flow problems only need to be solved in the Laplace space. The analytical equations are simpler and the drawdowns are easily evaluated by this method. When working with Laplace space solutions, it is also possible to consider more complicated flow problems.

Aquitards are often fine-grained sediments that are more compressible than coarse-grained aquifer materials, so they can potentially release large quantities of water from storage. In view of this, it is remarkable that it is commonly assumed that aquitard compressibility is negligible. Mathematical difficulties in solving storative aquitard problems seem to be the principal reason. Hantush (1960) was the first to take this storage effect into account for well flow in a single leaky aquifer. Moench & Ogata (1984) use the same problem as one of their examples. A Laplace space solution that allows for the effects of aquitard storage was subsequently developed for transient well flow in multi-aquifer systems. Compared to the previously developed transient solution, drawdowns could also be computed more easily. The existing computer program for pumping test analysis was upgraded for the new technique, rewritten in Pascal and compiled for a DOS personal computer.

More or less parallel with the above mentioned developments for multi-aquifer solutions, the Dutch hydrologist C. Maas was working on the same type of problems (Maas, 1986, 1987a,b). Personal communication (1986) showed that, independently, the same problem was solved using different numerical inversion techniques. This allowed a mutual check of the computed results and consequently a more general comparison of the used methods and their accuracies. The individual approaches, different algorithms and their comparable outcomes resulted in a joint publication in 1987 (Chapter 4).

Several years later Cheng & Morohunfola (1993) dealt with exactly the same problem and, unaware of the results published in 1987, derived a similar but less concise solution (Hemker & Maas, 1994; Cheng, 1994).

#### 4. NUMERICAL INTERMEZZO

With a method to analyze pumping tests in multi-aquifer systems, implemented in the computer program MLU (Multi Layer Unsteady state), the main problem was solved and there was no direct need for further development of analytical solutions. Over the years a growing number of consultants, mainly in the Netherlands, have used MLU for analysis of their pumping tests.

The increasing availability of personal computers, with their graphical output and easy interactive operation, provided a good opportunity to simplify many administrative steps that previously hampered numerical groundwater flow modeling. In 1986 it was decided to develop a new computer program for multi-aquifer modeling based on the finite-element technique (Hemker, 1988). The initial version of Micro-Fem appeared in 1987, and until 1996 a large number of improvements were implemented including transient flow, particle tracking and parameter estimation (Hemker & Van Elburg, 1990; Hemker & Nijsten 1996; Diodato, 1997). A Windows version is presently available and the free software, both MLU and MicroFEM, can be downloaded from the Internet (Hemker, 1999c).

The MicroFEM code was verified by comparing numerical results of radial multi-aquifer flow with those of MLU. More recently, some MLU results that conflicted with published drawdown curves for a partially penetrating well with wellbore storage were checked with a MicroFEM model to verify the MLU results (Chapter 5).

Since the present work is restricted to analytical solutions, MicroFEM developments and possibilities are not further discussed here.

## 5. TRANSIENT WELL FLOW SOLUTIONS FOR LAYERED AQUIFER SYSTEMS

### *The multi-layer well flow approach*

The term "layered aquifer system" is used to denote two types of aquifer system: a) a system of multiple aquifers, or multi-aquifer system, which comprises a series of aquifers separated by confining layers, and b) a vertically heterogeneous, stratified or multi-layered aquifer, which is a single aquifer composed of a number of sublayers. The analytical solution for multi-aquifer systems (Chapter 4) can be directly applied to multi-layered aquifers. This is actually a simplification, because transient flow in aquitards and aquitard storage need not be considered. Vertically heterogeneous aquifers can be treated in this way, but subdivision of a homogeneous aquifer into a large number of sublayers can also be useful, since this takes vertical flow within the aquifer into account. These vertical components of flow are approximated as finite-differences within the vertically discretized aquifer, while the horizontal, radial components are simultaneously solved analytically for all sublayers.

The concept of one-dimensional numerical modeling to account for vertical flow components in well flow problems was suggested in 1984 (Chapter 2) and further described with examples in 1985 (Chapter 3), but had not attracted much attention. In 1996 the conference on "Analytic-based modeling of groundwater flow" in Nunspeet (the Netherlands), provided a welcome opportunity to explore the interesting problem of well-face flow to a partially penetrating well, as known for example from the work of Haitjema & Kraemer (1988).

### *Vertical flow components and wellbore storage*

When transient well flow to a fully penetrating well in a fully confined homogeneous aquifer of constant thickness is considered, drawdown responses are independent of depth and there is only radial flow. However, vertical components of flow usually occur near the well caused by partial penetration of the well, by recharge from leakage through adjacent layers or from a declining water table and/or by heterogeneity. To investigate the capability of the multi-layer solution technique to accurately reproduce the effects of vertical flow, results were compared with several analytical well flow models including the delayed response model of Neuman (1972), the partial penetration solution of Moench (1993) and Székely's (1995) example of well flow in a heterogeneous model (Chapter 5).

A study of well-face flow requires an extension of the solution to include a finite well radius and, in the case of transient flow conditions, the effects of wellbore storage are an additional complication. The multi-layer solution presented in Chapter 5 is an extension of Chapter 4 by taking the well radius and wellbore storage into account. These features were implemented in the MLU computer program and the software was tested by comparing results with published graphs from Boulton & Streltsova (1976) and Narasimhan & Zhu (1993). Both cases showed deviations, but these could not be attributed to the present solution technique (Chapter 5).

### *The uniform well-face drawdown solution*

Analytical well flow solutions usually assume a uniform flux at the well face. In the case of a vertically heterogeneous aquifer this can be generalized as a "uniform well-face gradient" (UWG) condition. From the UWG multi-layer solution, presented in Chapter 5, it was clear that in all cases with vertical flow components, computed drawdowns within the finite-radius well varied from layer to layer. Instead of a uniform well-face flux or gradient, it is more realistic to assume a uniform well-face drawdown (UWD). From a mathematical point of view the transient UWD-condition is quite complex. Since the total well

production is given, while the uniform well-face drawdown is only a function of time, the resulting well-face flux is a function of both depth and time. There is no analytical solution known for this condition.

Multi-layer UWD solutions are presented in Chapter 6 for flow to partially penetrating, finite-diameter wells with wellbore storage and a thin skin (well-face resistance), for a constant and variable well discharge. The constant discharge case is further analyzed and examples are given of the depth and time dependent well-face fluxes. Drawdowns and fluxes are compared with UWG flow and with published results from numerical models (Ruud & Kabala, 1997). Differences between UWG and UWD flow are presented for various situations with vertical flow components including partial penetration, delayed response and heterogeneity (Chapter 6). The UWG and UWD conditions have been implemented as an option in the MLU computer program (Hemker, 1999c). An earlier version of Chapter 5 was presented during the conference on "Analytic-based modeling of groundwater flow" (Hemker, 1997), and both Chapters 5 and 6 have now been submitted for publication.

## 6. OUTLINE OF PRESENTED ANALYTICAL SOLUTIONS

The present work comprises five individual papers. These may be divided into two parts: the first comprises Chapters 2 through 4, in which aquifer hydraulics for fully penetrating wells in multiple aquifer systems are presented. The second part contains Chapters 5 and 6, and deals mainly with large-diameter, partially penetrating wells in multi-layered aquifers.

Chapter 2, the first paper, can be read independently of the others. Each succeeding chapter builds on the previously published results, presented in the preceding chapters. This gradual development of the theory concerns both the applied techniques and the ongoing complexity of the considered well flow models. An outline of this evolution is presented as a list of well flow characteristics and mathematical methods, arranged in order of appearance in consecutive chapters.

### Chapter 2:

- Leaky multiple aquifer system
- Steady-state flow
- Fully penetrating pumping well
- Eigenvalue decomposition of the system matrix

### Chapter 3

- Transient well flow
- Unconfined conditions
- Stratified aquifers
- Laplace transformation
- Hankel transformation

### Chapter 4

- Storage of the aquitards
- Dual and triple porosity models
- Numerical inversion of Laplace transformation

### Chapter 5

- Multi-layered aquifer
- Vertical heterogeneity
- Finite-diameter well
- Wellbore storage
- Partial penetration
- Generalized delayed response

### Chapter 6

- Variable discharge
- Slug test
- Skin effect
- Uniform well-face drawdown
- Variable well-face flux

To use the presented analytical solutions as a method for analysis of aquifer tests, computer software is required. As far as a constant well discharge is concerned, all mentioned well flow characteristics are implemented in the computer program MLU, which can be run on every DOS-compatible computer. A three-aquifer version is available from the Internet (Hemker, 1999c).

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## SUMMARY

The focus of this work is deterministic models of saturated groundwater flow in layered aquifer systems. The term "layered aquifer system" is used to denote two types of system: a) a system of multiple aquifers or multi-aquifer system which comprises a series of aquifers separated by confining layers, and b) a vertically heterogeneous, stratified or multi-layered aquifer which is a single aquifer composed of a number of sublayers. Analytical solutions for many different types of well flow in single aquifers are known and used for pumping test analysis, while only few solutions for two-aquifer systems are available. Pumping tests in the Netherlands have shown that sometimes four or more aquifers are involved. The lack of a proper solution for multi-aquifer systems was felt as an omission in well flow theory. The main objective of this study, therefore, was to develop analytical solutions that can be used for determining geohydrological properties from pumping tests in layered aquifer systems. The study was carried out in two separate periods. Flow in multi-aquifer systems was investigated during the years 1979 to 1986 and, more recently, the same solution method was further developed to investigate flow near wells in multi-layered aquifers.

Groundwater modeling always requires simplification of the more or less complex geology. Pumping tests analysis techniques are mainly based on analytical solutions and the conductive and storative characteristics of each layer are assumed to be homogeneous. At the scale of a pumping test a multiple aquifer system, or a multi-layered aquifer, will be closer to reality than the classical single, homogeneous aquifer models. Generally speaking, an increasing number of layers allows better representation of the aquifer system, but the number of parameters grows proportionally. This vertical discretization of the flow domain gives multi-layer solution techniques a position between the analytical and numerical solution methods. The multi-layer approach can be very useful, especially at the scale of well-flow problems, since it combines the best of both worlds.

The present work is a compilation of five interrelated papers, presented sequentially in Chapters 2 to 6 and preceded by a general introduction (Chapter 1). The papers may be divided into two parts: the first comprises Chapters 2 through 4, in which aquifer hydraulics for fully penetrating wells in multiple aquifer systems are presented. The second part consists of Chapters 5 and 6, which deal mainly with large-diameter and partially penetrating wells in multi-layered aquifers. Chapter 2, the first paper, can be read independently, but each succeeding chapter builds on the previously published results. This gradual development of the theory concerns both applied techniques and the ongoing complexity of the considered well flow models.

A general method is developed in Chapter 2 for an analytical solution of steady flow problems in leaky multiple aquifer systems. This is based on the assumptions that only horizontal flow in the aquifers and vertical flow in the aquitards need be considered, a common simplification in leaky aquifer theory. The method is characterized by the construction of a tridiagonal system matrix and decomposition of this matrix into its eigenvalues and eigenvectors. The solution has been applied in the derivation of analytical solutions for parallel flow to canals and radial flow to wells. One of the examples presents results from a pumping test in a six-aquifer system.

The solution method for steady-state flow is further developed in Chapter 3 to obtain exact analytical solutions for transient well flow problems in leaky and confined multiple aquifer systems. Equations are presented for the drawdown distribution in systems of infinite extent, caused by wells penetrating one or more of the aquifers completely and discharging each layer at a constant rate. The

possible use of these solutions to approximate more complex systems is discussed and illustrated by examples of aquitard storage, unconfined conditions, stratified aquifers and partially penetrating wells.

Chapter 4 deals with the search for relatively fast and sufficiently accurate computing methods to determine the drawdown in all aquifers of a multiple aquifer system. Efficient solutions are found by using the Laplace transform and different numerical Laplace inversion methods. Another advantage of working with a solution in the Laplace domain is that the effects of elastic storage of the aquitards can be incorporated in the mathematical formulation. The similarity between layered aquifer solutions with linear or diffusive crossflow and double-porosity models for fissured formations is used to demonstrate how more complex multiple-porosity layered formations can be modelled by constructing an appropriate system matrix.

Solutions for multi-layered aquifers are introduced in Chapter 5. The Laplace domain solution from Chapter 4 is extended to include the effects of a finite-diameter well and wellbore storage. Vertically heterogeneous aquifers can be handled, but the subdivision of a homogeneous aquifer into a large number of sublayers is shown to be also useful, since it takes vertical flow within the aquifer into account. These vertical components of flow can be caused by: a) partially penetrating wells, b) recharge from leakage through adjacent layers or from a declining water table, and c) heterogeneity. To obtain comparable results using analytical solutions for three-dimensional transient well flow, a uniform head gradient is assumed at the well screen: the uniform well-face gradient (UWG) condition. Instantaneous and delayed drainage of water from above the water table are considered, combined with the effects of partially penetrating and finite-diameter wells. Other applications demonstrate the transient effects of wellbore storage in unconfined aquifers and the effect of decreasing specific storage values with depth in an otherwise homogeneous aquifer. Special attention is paid to the systematic drawdown error near the end of partially penetrating well screens, which can be approximated as a function of sublayer thickness.

In order to deal with transient well flow with the more complex condition of a spatially constant head along the well screen, a further extension of the multi-aquifer solution method is developed in Chapter 6. The resulting uniform well-face drawdown (UWD) Laplace domain solutions are proposed for well flow caused by constant, variable and slug discharges and also include the effects of a finite diameter well, wellbore storage and a thin skin. Since no analytical solutions are known for transient UWD well flow, wellbore drawdowns, well-face flux distributions and aquifer drawdowns are compared with published numerical results.

The differences between UWG and UWD well flow solutions for a constant discharge rate were studied for: a) partially penetrating wells in confined aquifers, b) fully penetrating wells in unconfined aquifers with delayed response and c) layered aquifers and leaky multi-aquifer systems. For a partially penetrating, finite diameter well in a confined aquifer it has been shown that the UWD model causes lower well drawdowns than the UWG model. The well-face flux distribution is obtained as it develops in time and attention is paid to well screens of different lengths and diameters. The difference between UWG and UWD flow toward a fully penetrating well in an unconfined aquifer shows that the transient well-flux distribution is clearly nonuniform during intermediate times. The relatively large discharge from the upper part of the screen resulted in a more rapid lowering of the water table near the well. Finally, the UWD solution is applied to various types of layered aquifers and multi-aquifer systems in order to analyze the well flux distributions and drawdowns as a result of heterogeneity, aquitard storage and recharge through a leaky top.

The capability to model multiple aquifer systems and vertically heterogeneous aquifers without the need to construct transient numerical models makes the presented solutions particularly suitable for well tests, flowmeter tests, slug tests and pumping tests. A DOS-based computer program for UWG and

UWD constant discharge pumping test analysis is available from the author, while a three-aquifer version is available from the Internet.

## **ABOUT THE AUTHOR**

Christiaan (Kick) Johan Hemker was born in the Hague on May 6th 1945. He attended the Comenius Lyceum (HBS-B) in Amsterdam and in 1963 started a career as ship's officer in the merchant navy. After sailing the high seas for many years, a new start was made in 1971 by choosing Physical Geography at the University of Amsterdam (UvA) as an appropriate study. In 1975 he graduated (kandidaats) with mathematics as a subsidiary subject and switched to Hydrology at the Vrije Universiteit (VU). With a primary interest in groundwater hydrology, he started field investigations with the "A & V" Water Supply Company in 1978, where the present research into "multiple-aquifer systems" was first encountered. In February 1980 he graduated at the VU (cum laude) and remained as a part-time groundwater hydrologist with the Water Supply Company until 1991. From 1982 to 1986 he was employed part-time by the Vrije Universiteit as a lecturer in groundwater hydraulics. When Professor N.A. De Ridder retired in 1990, he was appointed as part-time lecturer in groundwater modeling and the following year this was extended to also teach groundwater hydraulics. Additional professional activities include acting as a groundwater consultant and the development and marketing of groundwater software. The MicroFEM package, designed for finite-element, multiple-aquifer modeling of transient groundwater flow, is undoubtedly one of his most prominent products and a completely new Windows version of MicroFEM has only recently been released.

Kick Hemker  
hemker@microfem.com  
<http://www.microfem.com>